MATHEMATICAL PREPARATION DESIRED FOR HIGH SCHOOL PHYSICS.*

By Mr. Clifford S. Griswold.

Physics teachers are not particularly united either in their method of teaching, in the selection of topics, in emphasis on the various parts of the topics, or even the object to be attained by their teaching, but there is one point on which they all agree: that the relation between mathematics and physics is very unsatisfactory.

Pupils who have done well in their mathematics seem to be all at sea when they reach physics. Principles learned and applied with facility in the problems and equations of mathematical textbooks are beyond their power to apply in this new field. Computation is labored and beset with errors,—many simple operations requiring repeated explanation notwithstanding the training in arithmetic.

In geometry, lines, surfaces, angles, etc., are studied constantly, and their relations thoroughly investigated, but not a pupil enters the physics laboratory with any idea of accurate measurement of these familiar things, nor has he first-hand knowledge that these actually demonstrated facts are true in practice.

The physics teacher, given what seems to him a very meagre allowance of time in the school course to teach the physics that he is required and wants to teach, feels that it is quite impossible for him to take the time to teach the pupil the mathematics he doesn't know, and to adapt the mathematics he does know, in short, to make him over mathematically.

He has before him two alternatives, either to get along without mathematics and pay more attention to lecture table demonstration, dispensing with the problem book, or to change the mathematics teaching so that it shall coördinate more helpfully

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with the physics. If he chooses the former he finds that after a time his progress is arrested and results can only be obtained by the aid of mathematics, for no real progress in science is possible without mathematics. Then comes the time that the teacher of physics comes up against the fact that the mathematics as received by the pupils in their early training is not suited for his use and he has now to consider the difficult matter of adapting or changing the early training so as to correlate more naturally with his subject. Now it is a truism in education that the concrete must come before the abstract and it is too often a matter of sad experience that many of our troubles come from requiring pupils to deal in abstractions before they are ready for them.

It is coming to be realized more and more that the old-fashioned home work, chores, farm work, hand work of all descriptions furnished a ground work of concrete experience, of which the value, importance and necessity have only lately been propperly understood, as they provided a foundation of self education on which the structure of theory and abstract reasoning could be built surely and swiftly.

Mr. Edison is a brilliant example of the aid of concrete experience to mathematical or quantitative comprehension. With little knowledge of the theory of mathematics, beyond some arithmetic and almost no algebra, he yet can make quick complicated mathematical deductions by a sort of instinctive inference from his vast store of accumulated facts and experiences.

Few have this ability to the degree that Mr. Edison has, but every one knows that the more one's knowledge of practical relations of quantity, dimension, etc., is, the less likely one is to go astray in abstract theory, as he has always at hand the corrective presence of observed fact and experience to act as a check against possible error and misunderstanding.

It seems clear then that one of *the* great difficulties the mathematics teacher—that all teachers—meet with, is the lack of concrete experience on the part of the pupils.

The physics teacher was right to emphasize his laboratory work, his lecture table experiments, to insist upon examining, taking apart, putting together of fixtures, machines and apparatus, but he should have done this not as a *substitute* but as a *foundation* for mathematics.

I was asked by your president to discuss the subject of preparation in mathematics desirable for high school physics. In general, it seems to me that the best preparation might be formulated on the following lines, namely, to have the subject of mathematics conceived and presented to young pupils, as a means of quantitative expression of the relations of various laws, phenomena, etc., of which the pupils have had immediate experience in some striking concrete fashion, either through the use of models, diagram, actual measurement or experiment; rather than to conceive and present mathematics as an end in itself or to follow any such course as to permit of its being mistakenly so conceived.

That is, rather than begin the study of a new mathematical subject or process, as an unrelated, detached subject, to create conditions through the apparatus, model, or experiment which the new mathematical process is necessary to explain.

If mathematics could be so presented as a means to a definite and where possible physical and concrete end—and I know that this is an extremely difficult matter and one calling for extensive in the present method, program, and apparatus of mathematics teaching—physics could ask no more of mathematics teaching.

The pupil would come to physics not as to a strange subject offering violent contrasts and forced relations to his mathematics, but into a subject in which he feels his mathematics will permit him to penetrate more deeply. A subject of which he has already curiosity—and interest—stimulating glimpses, and which he already knows something about from the contributions that physics has made to the problems, situations, experiments, etc., for which he has had to study his mathematics to be able to deal with, and best of all, he will have learned that his mathematics is a tool for use and not an *accomplishment* for exhibition or passing examinations well.

How the mathematics course shall be planned in detail to meet this requirement I am afraid I am not wise enough to say. In the beginning at least, this question would be answered variously according to the opportunities for collecting this material or the particular bent of the mathematical teacher's mind.

At Booker Washington's school at Tuskegee, every new build-

ing put up furnishes an astonishing amount of original material for geometry.

At Groton, I loaned the simpler physical appartus to the mathematics teacher as one item of material for quantitative expression.

It would be well to introduce frequent exercises involving actual construction and accurate measurement,—which would lead naturally to drawings to scale—and show reason for accurate computation.

Make use of live, instructive, interesting problems. I find this in a recent algebra, "Five years ago a father was six times as old as his son and five years hence the father will lack five years of being three times as old as his son will be at that time. What are their ages?" Contrast that with the following, "Assuming the velocity of sound to be 1,100 ft. a second, find the distance of the point of discharge if 24 seconds elapse between seeing the lightning and hearing the thunder."

The first problem is not a good vehicle for the mathematics it illustrates, for it has been used to secure a bit of knowledge which in all probability the eager minded pupil would regard with considerable contempt.

The second problem might very well stimulate the pupil to try the experiment for himself. In any case it would be an introduction to a line of phenomena which he would therefore be willing and able to learn more about later.

Popular knowledge and reading abounds in references to *candle power, amperes, velocities, temperature, horse power, machines, lenses, mirrors,* etc., in which all pupils are interested and an understanding of which is most meagre without some mathematics—furthermore the work of *engineers, electricians, surveyors, navigators, mechanics, contractors,* etc., contain materials for countless problems on subjects of familiar interest.

The study of graphs is useful in many ways to the mathematics teacher, in giving the pupil a better understanding of linear, simultaneous and quadratic equations. I have known many cases of boys failing to grasp the principles of simultaneous equations until they had seen the graphs *intersect* when it became perfectly clear to them.

The number and kinds of solution possible are more clearly

shown and understood through the graph than by theoretical proofs, which are more bewildering than helpful to the average pupil.

This knowledge of graphs is of great service to the pupil when he reaches physics, as experimental results in physics may often be expressed in the form of a graph.

Says John Perry: "The basis of all application of mathematics is the fact that any physical phenomenon which is directional, such as a force, a velocity, a stress, the flow of a liquid, etc., may be expressed by a straight line, and many other relations are expressed by curved lines, such as Boyle's law, capillarity, the pendulum, elasticity, the polarization of a voltaic cell, etc. Much of this data too can be used for plotting in the earlier mathematics.

It is generally true that the teacher of physics is obliged to teach whatever geometry is needed for his subject. The pupil beginning physics very soon encounters the composition of velocities and forces in the text, and soon again in the laboratory, but yet in spite of his earlier training in geometry, he has to be shown how to complete the parallelogram to find the resultant of the component forces.

Reflected motion, stability, curvilinear motion, pendulum, lever, inclined plane, wave motion, images in mirrors and lenses, and index of refraction of light are topics requiring a workable knowledge of constructional geometry, and these subjects again furnish data for the constructional problems of the earlier geometry, thus involving, in a purposeful way, work in lines, angles, perpendiculars, parallels, triangles, squares, and circles using cross-section paper, protractors, dividers, diagonal scale, and the metric rule.

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